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Modelling drum columns with discrete elements – Practical issues

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Outline

- Discrete elements
  - 3DEC and related approaches
- Blocks
  - Rigid, Deformable, Time steps
- Contact discretization
- Joint stiffness
- Block breakage
- Reinforcement elements
  - Local reinforcement, User-programmed elements
- Damping
  - Mass, Stiffness, Maxwell

Discrete elements (DEM)

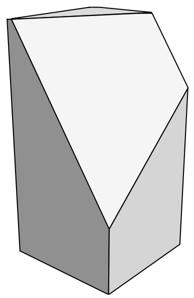
- DEM is a class of numerical methods for discontionuous structures with many different formulations:
  - **DEM, RBSM, FDEM, NSCD, AEM, …**

- This presentation refers mainly to Cundall’s approach (e.g. 3DEC) and related formulations characterized by:

  - Blocks: Rigid or Deformable (internal FEM mesh)
  - Point contacts
  - Large displacements with geometry and contact updates
    - Essential for large amplitude rocking
  - Dynamic analysis with explicit time stepping algorithms
    - Requires small time steps
    - Robust for strongly nonlinear problems and geometry/contact updates

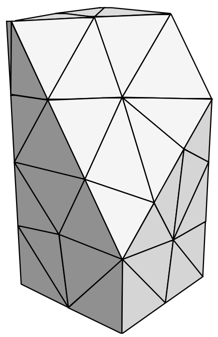
DEM – Block representation

- Rigid blocks
  - Block moves as a rigid body
  - All system deformability lumped at the joints (joint stiffness parameters)
  - Computational efficient for dynamic analysis



rigid polyhedral block

- Deformable blocks
  - Internal finite element mesh
  - Internal stress state
  - Elastic or non-elastic behaviour



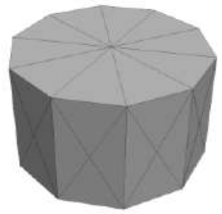
deformable block with internal mesh of tetrahedra

- Time step
  - Rigid blocks $$\Delta t = 2\sqrt{\frac{m^*}{k^*}}$$
  - Deformable blocks $$\Delta t < \frac{h}{c_P}$$

Example

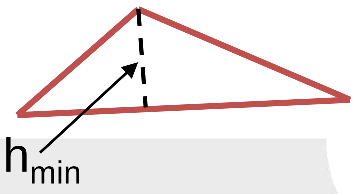
Parthenon column (Psycharis et al. 2003)

- Rigid block time step
  - Kn = 1 GPa/m
  - Δt = 2.8e-4 s



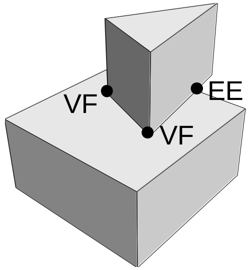
- Deformable block time step
  - Zone (element) edge = 0.5 m
  - E = 30 GPa
  - Δt = 2.3e-5 s

Time step governed by smallest tetrahedral height, difficult to control in standard mesh generation



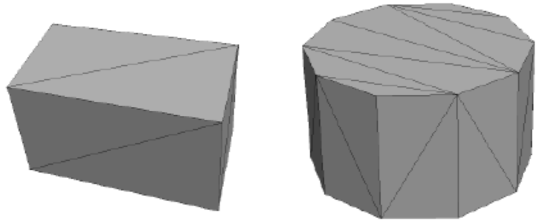
Rigid blocks – Contact discretization – Face triangulation

- The triangulation of the rigid block faces governs the number of contact points located at
  - Vertex-face locations
  - Edge-edge locations

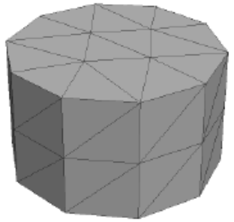


- Options (in 3DEC)
  - Default triangulation
  - Edge-max e
  - Radial
  - Radial-8 (e.g., for bending of quadrilateral cross-sections)

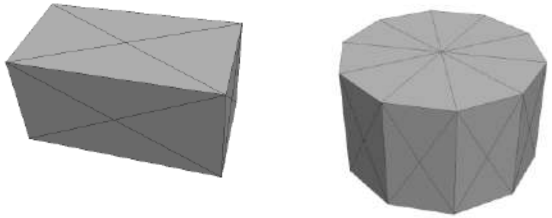
- Default triangulation



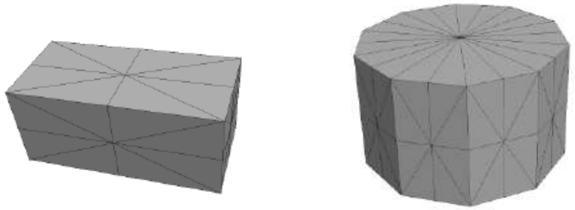
- Edge-max 0.5



- Radial



- Radial-8



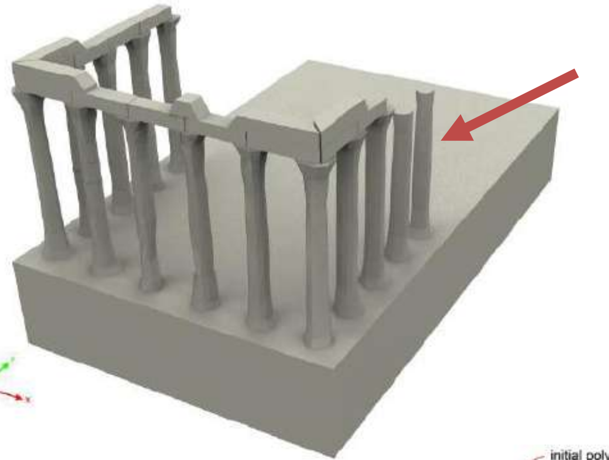
Joint stiffness

- Rigid block models: joint stiffness parameters (Kn, Ks) represent the global structural deformability
- For stone block masonry with dry joints and stiff units, a substantial amount of deformability may be due to the joints (irregularity, non-planarity, damage)
- Natural frequencies provide a measure of the in situ deformability, and may be used to estimate joint stiffnesses

Example

Roman temple, Évora, Portugal

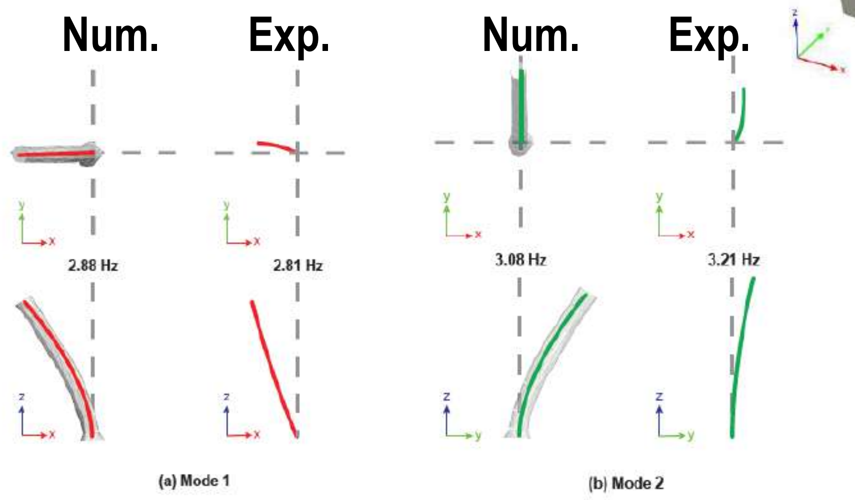
(Nayeri 2012; Oliveira et al. 2014)

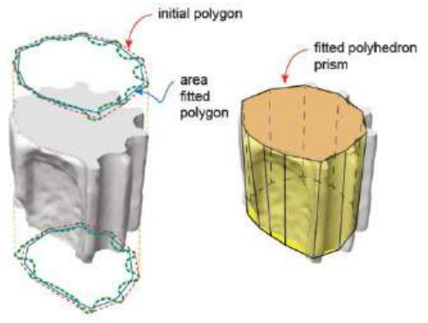


Free standing columns

Column E6 – Frequencies (Hz)

Mode	Exp.	Num.
1	2.81	2.88
2	3.21	3.08

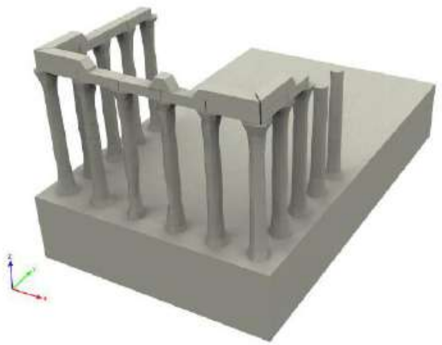




Geometric modelling of damaged drums

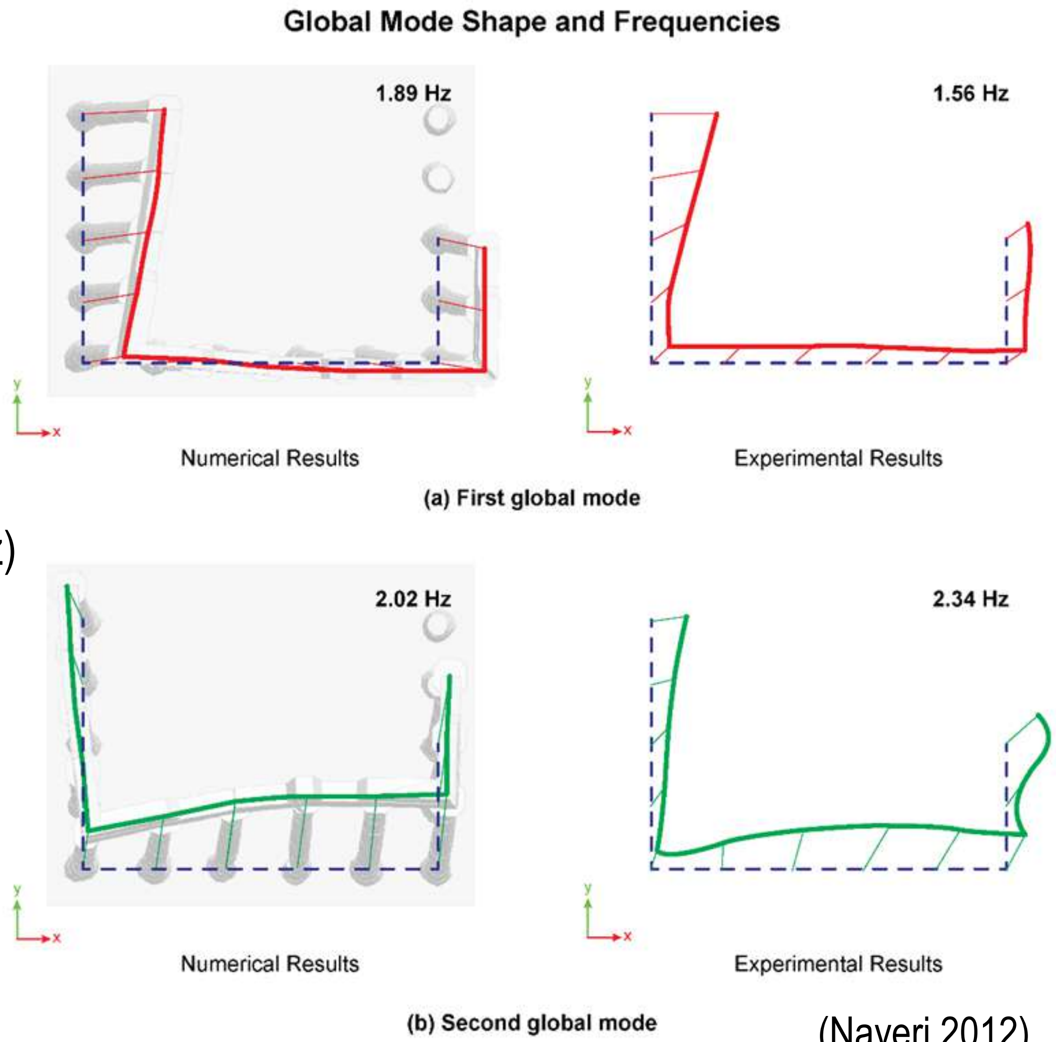
Natural frequencies – Global modes

- Joint stiffnesses estimated from measured frequencies of free-standing columns
  - Kn = 2.95 GPa/m
  - Ks = 1.47 GPa/m



Global modes – Frequencies (Hz)

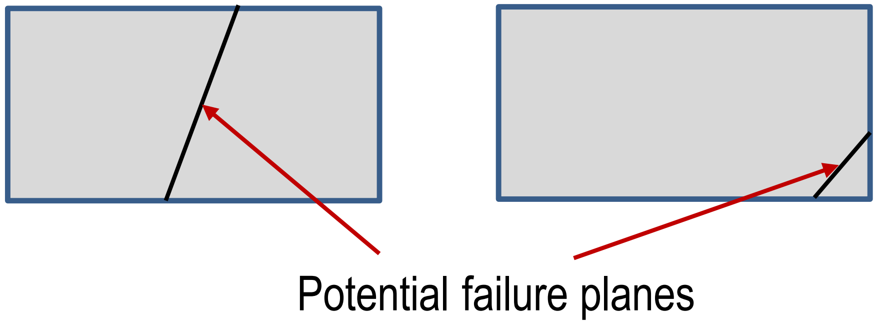
Mode	Exp.	Num.
1	1.56	1.89
2	2.34	2.02



(Nayeri 2012)

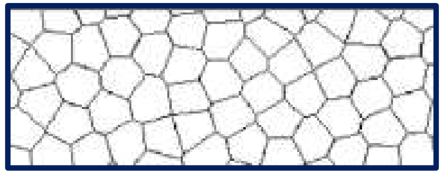
Block fracture / breakage

- Deformable block models with non-elastic materials
  - Computational demanding
- Bonded-block models - Simplified simulation of block fracture in rigid block models
  - Insert potential fracture planes in blocks
  - Contacts are assigned the cohesive and tensile strength of the block material
  - Block splitting simulated by failure of contacts



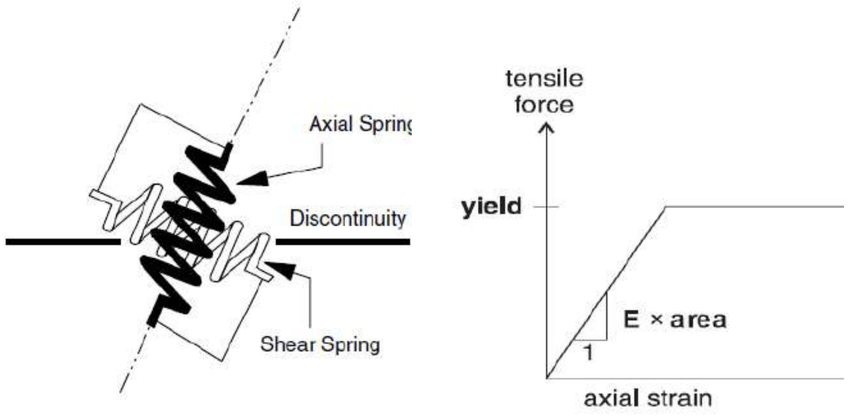
- More refined models may be created using random Voronoi networks of potential failure planes (mostly for static analysis)

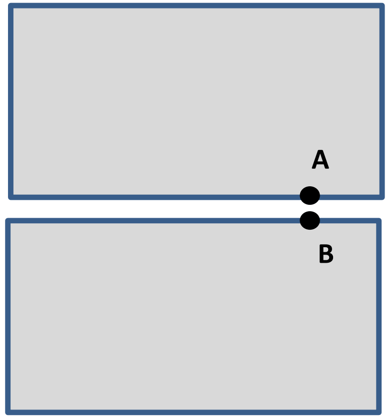
  (Sarhosis & Lemos 2018; Pulatsu et al. 2020)



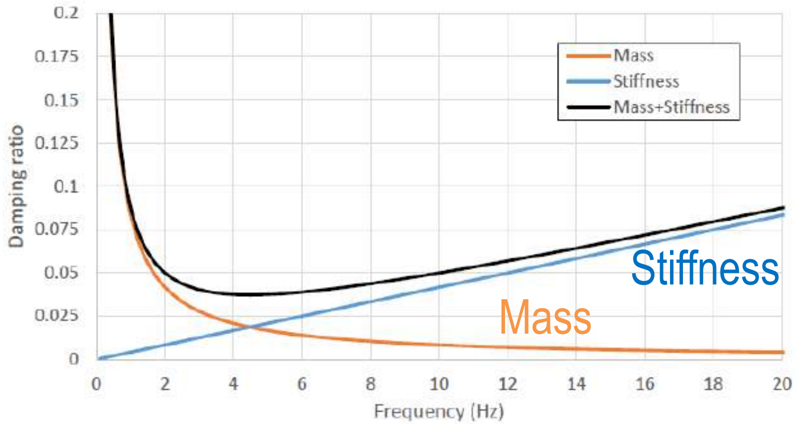
Reinforcement elements

- **Options in 3DEC**
  - Beam-type elements
    - Small time steps due to structural nodal masses
  - Local reinforcement – Connects 2 blocks across a joint
    - Represented by elasto-plastic elements (yielding / breakable springs acting in the normal or shear directions
  - User-programmed reinforcement elements
    - Connection between 2 blocks
    - Constitutive model programmed by user (Fish or Python)
      - Input: Relative movement between connection points A, B
      - Output: Forces to be applied to blocks





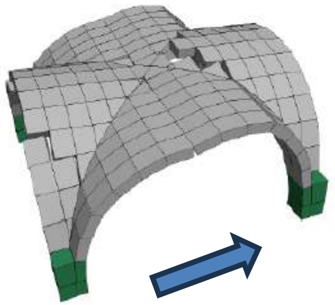
Damping – Rayleigh



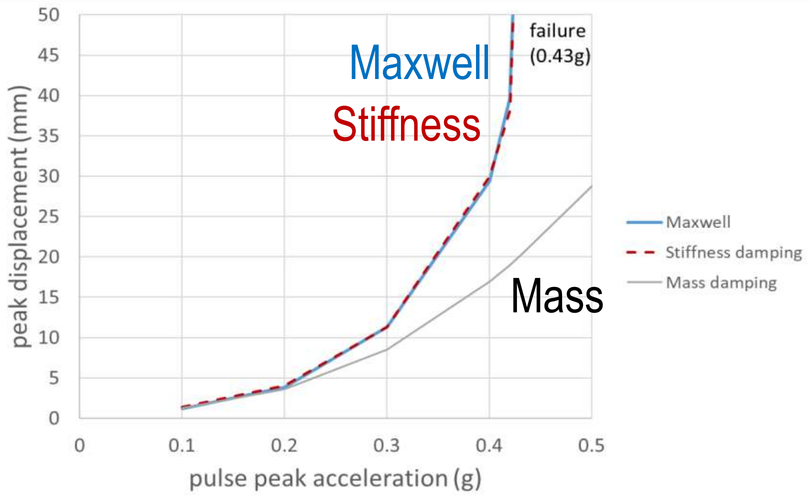
Rayleigh damping

Energy dissipation

- Constitutive model provides part of the energy dissipation
    - Frictional sliding; Cohesive/Tensile strength softening
- Some amount of viscous damping usually required to match field data



Dynamic pulse



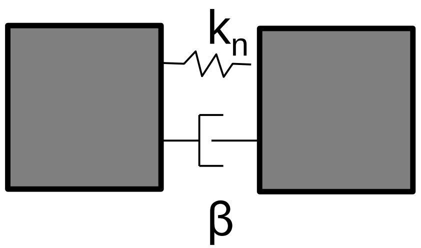
- **Rayleigh - Mass-proportional component**
    - Low values typically used
    - May affect failure modes (low frequency mechanisms)

- **Rayleigh - Stiffness-proportional component**
    - Physically meaningful
    - Requires a time step reduction in explicit algorithms

$$\Delta t < \frac{2}{\omega_{max}}\left(\sqrt{1+\xi_{max}^2}-\xi_{max}\right)$$

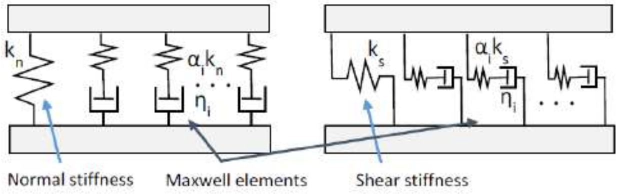
(reduction factor)

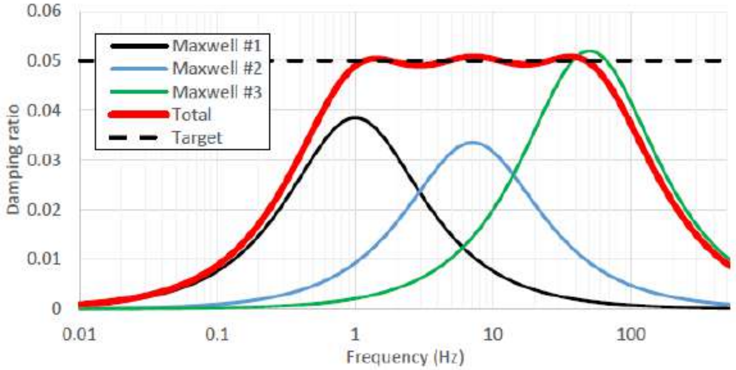
ξ - fraction of critical damping at highest frequency



Damping – Maxwell elements

- Maxwell damping / Frequency range damping
  - Rigid block model: elements applied at joints
  - Near uniform damping over a given range
  - Efficient for explicit algorithms
- Drawback: The spring in Maxwell elements causes a small increase in frequencies
- Advantage: Provides performance close to stiffness damping without time step penalty





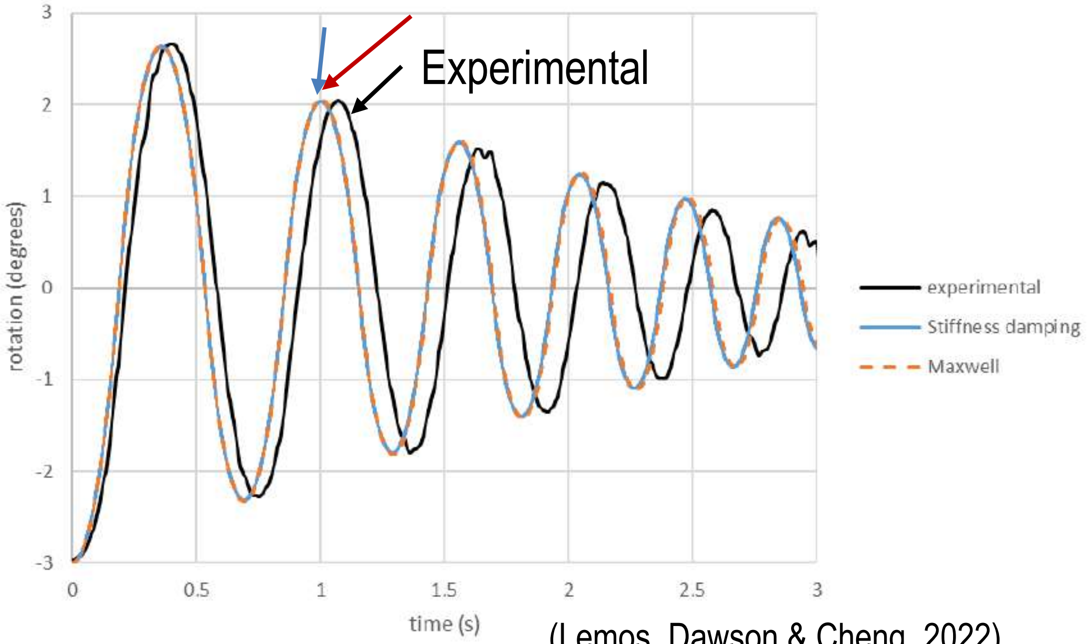
3 Maxwell elements

Block rocking experiments
(Peña et al. 2007)

Kn = 10 GPa/m

Sfiffness damping: $\beta = 8 \times 10^{-5}\ s$

Maxwell damping: 6% in 5-250 Hz



(Lemos, Dawson & Cheng, 2022)

Thank you